MA 161: Lesson 25

Graphing Functions - Part II (4.4)

Review of guidelines for sketching functions

More examples on sketching functions

Coming up: Optimization Problems

Office Hours

M, W, F: 2:45pm - 4:15pm

Sketching Guidelines $\rightsquigarrow$ $y = f(x)$

* Domain $\rightsquigarrow$ all possible inputs

* $\underbrace{x\text{-intercepts}}_{f(x)=0}$ and $\underbrace{y\text{-intercepts}}_{y=f(0)}$

* Vertical Asymptotes $\rightsquigarrow$ $\lim_{x\to a^+} f(x) = \pm\infty$, $\lim_{x\to a^-} f(x) = \pm\infty$

* End Behaviour $\rightsquigarrow$ $\lim_{x\to\pm\infty} f(x)$ $\left.\right\}$
  * H.A
  * Slant Asymptote
  * $\pm\infty$, No Asymptote

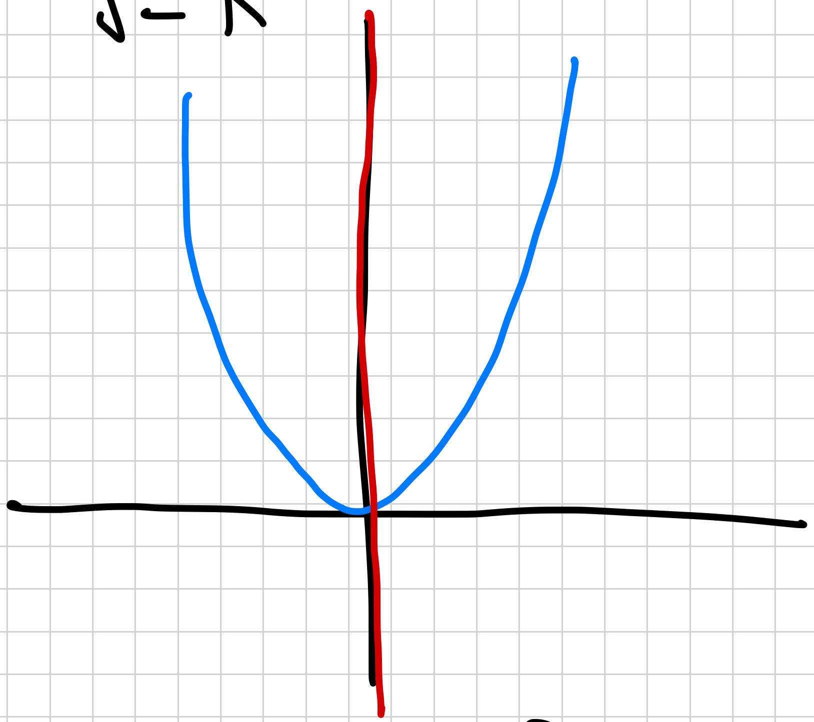
* $f'(x)$ $\rightsquigarrow$ $f' > 0$ $\uparrow$
  $f' < 0$ $\downarrow$

* $f''(x)$ $\rightsquigarrow$ $f'' > 0$ $\cup$
  $f'' < 0$ $\cap$
  $f'' = 0$ $\rightsquigarrow$ could have inflection point

* Symmetry

Symmetry

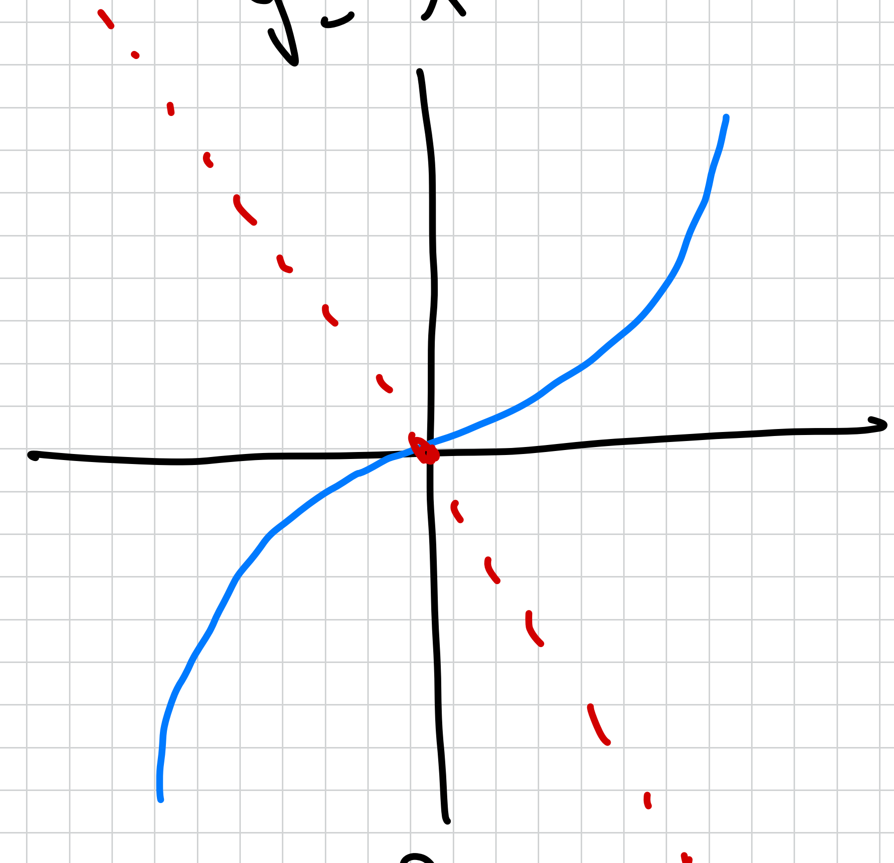
$y = x^2$



- Y-axis Symmetry
- even function

$$f(-x) = f(x)$$

$y = x^3$



- Origin Symmetry
- Odd functions

$$f(-x) = -f(x)$$

eg: Sketch the graph of $f(x) = e^{-x^2}$

Domain: $(-\infty, \infty)$

Symmetry: $f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$ $\Big\}$ even fn — y-axis symmetry

x-intercepts: $f(x) = e^{-x^2} = 0 \Rightarrow$ No x-intercept

y-intercepts: $y = f(0) = e^{0} = 1$

Vertical Asymptote: No V.A

End Behaviour

$$\lim_{x\to+\infty} e^{-x^2} = \lim_{x\to+\infty} \frac{1}{e^{x^2}} = 0$$

$y=0$ is a H.A

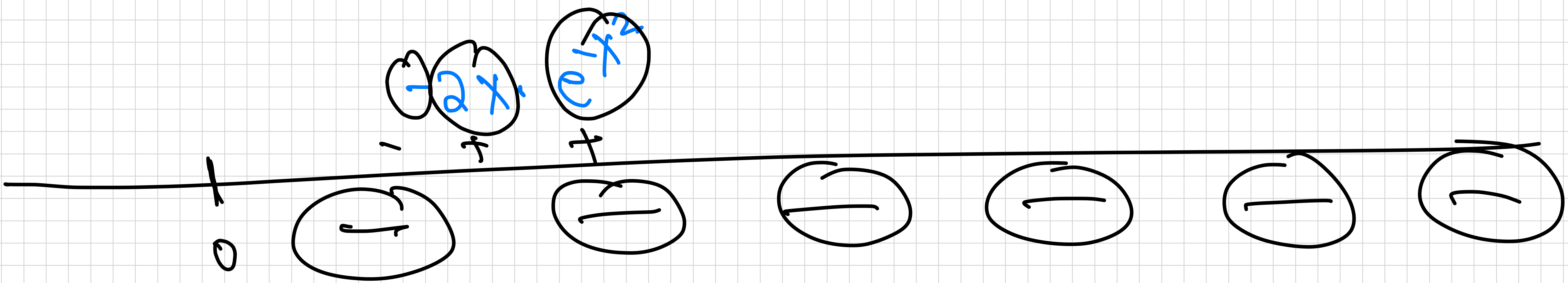
$$f(x) = e^{-x^2}$$

$$f'(x) = \frac{d}{dx}\left[e^{-x^2}\right] = e^{-x^3} \cdot (-2x)$$

Critical points:

$$-2x \cdot \underbrace{e^{-x^2}}_{>0} = 0$$

$x = 0$ is the only critical point



$f(x)$ is $\downarrow$

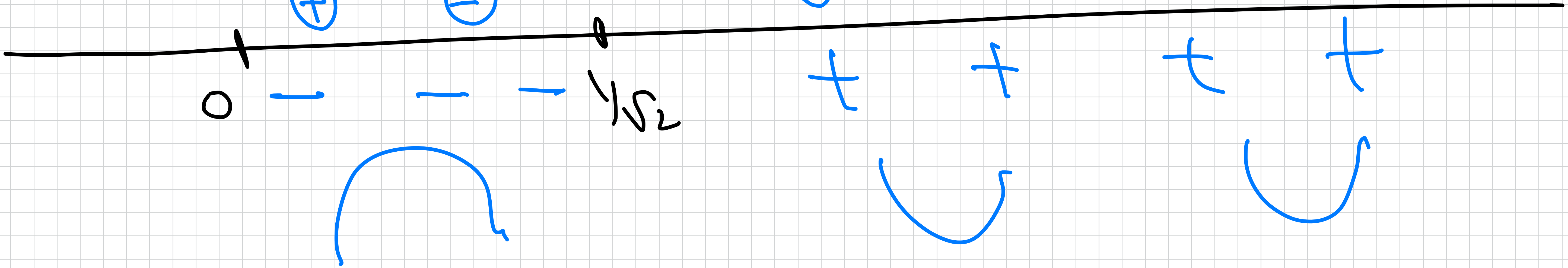
$f(x) = e^{-x^2}$, $\quad f'(x) = -2xe^{-x^2}$

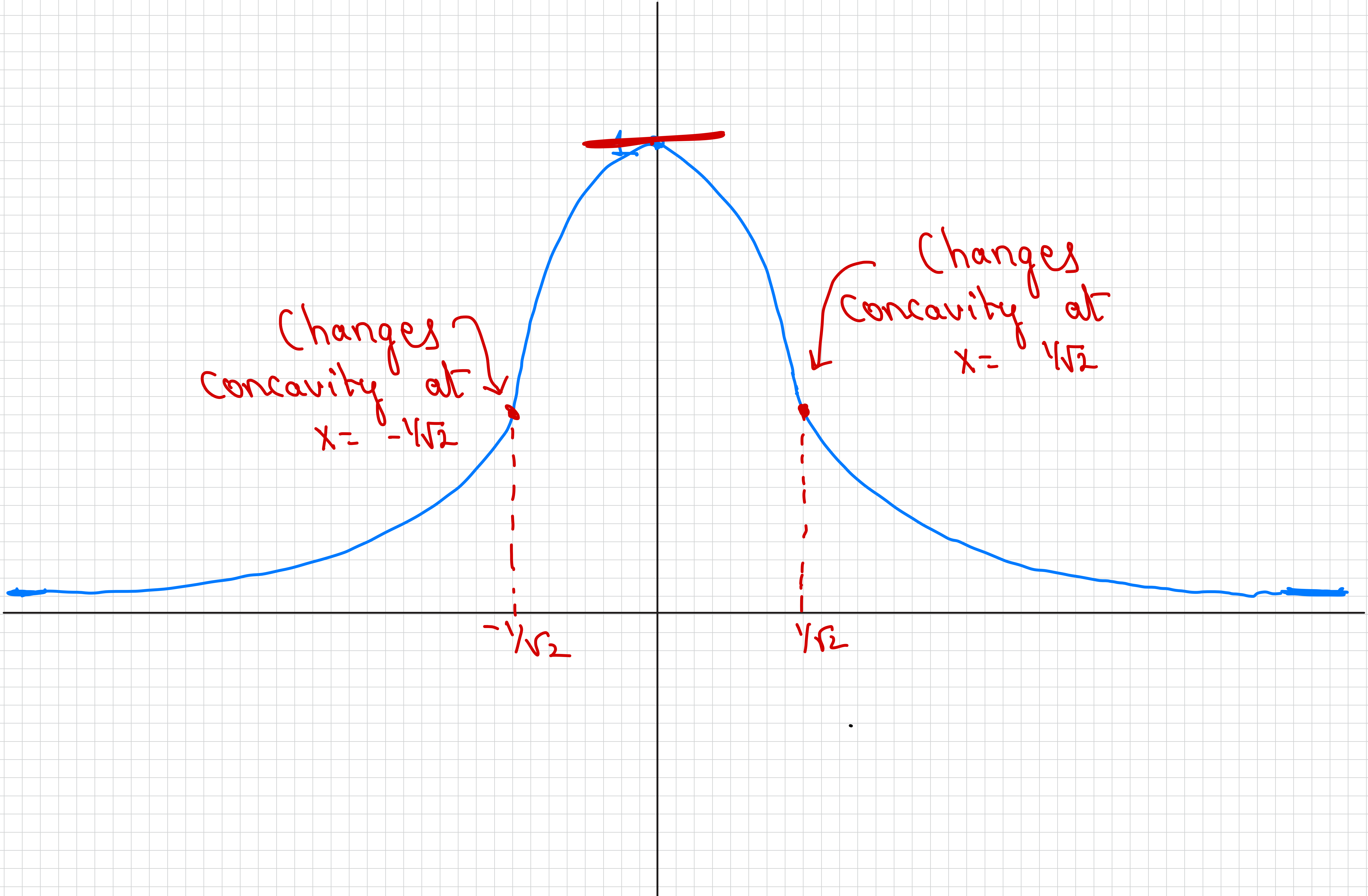
$$f''(x) = \frac{d}{dx}\left(-2xe^{-x^2}\right) = -2e^{-x^2} - 2x \cdot \frac{d}{dx} e^{-x^2} = -2e^{-x^2} + 4x^2 e^{-x^2}$$

$$f''(x) = \underbrace{2e^{-x^2}}_{>0}\left[2x^2 - 1\right] = 0 \rightsquigarrow 2x^2 - 1 = 0$$

$$x^2 = \frac{1}{2} \rightsquigarrow x = \frac{1}{\sqrt{2}}, \quad x = \frac{-1}{\sqrt{2}}$$

$$\underbrace{2e^{-x^2}}_{\oplus}\underbrace{(2x^2-1)}_{\ominus} \qquad \underbrace{2e^{-x^2}}_{\oplus}\underbrace{(2x^2-1)}_{\oplus}$$





Changes Concavity at $x = -1/\sqrt{2}$
Changes Concavity at $x = 1/\sqrt{2}$
$-1/\sqrt{2}$
$1/\sqrt{2}$

eg: Sketch the graph of $y = \sin x - x$ on $[2\pi, 2\pi]$

Symmetry: $f(x) = \sin x - x$

$f(-x) = \sin(-x) - (-x) = -\sin x + x$

$= -[\sin x - x] = -f(x)$

ODD function

x-intercept: $\sin x - x = 0 \to \boxed{\sin x = x}$?

y-intercept: $y = \sin(0) - 0 = 0$

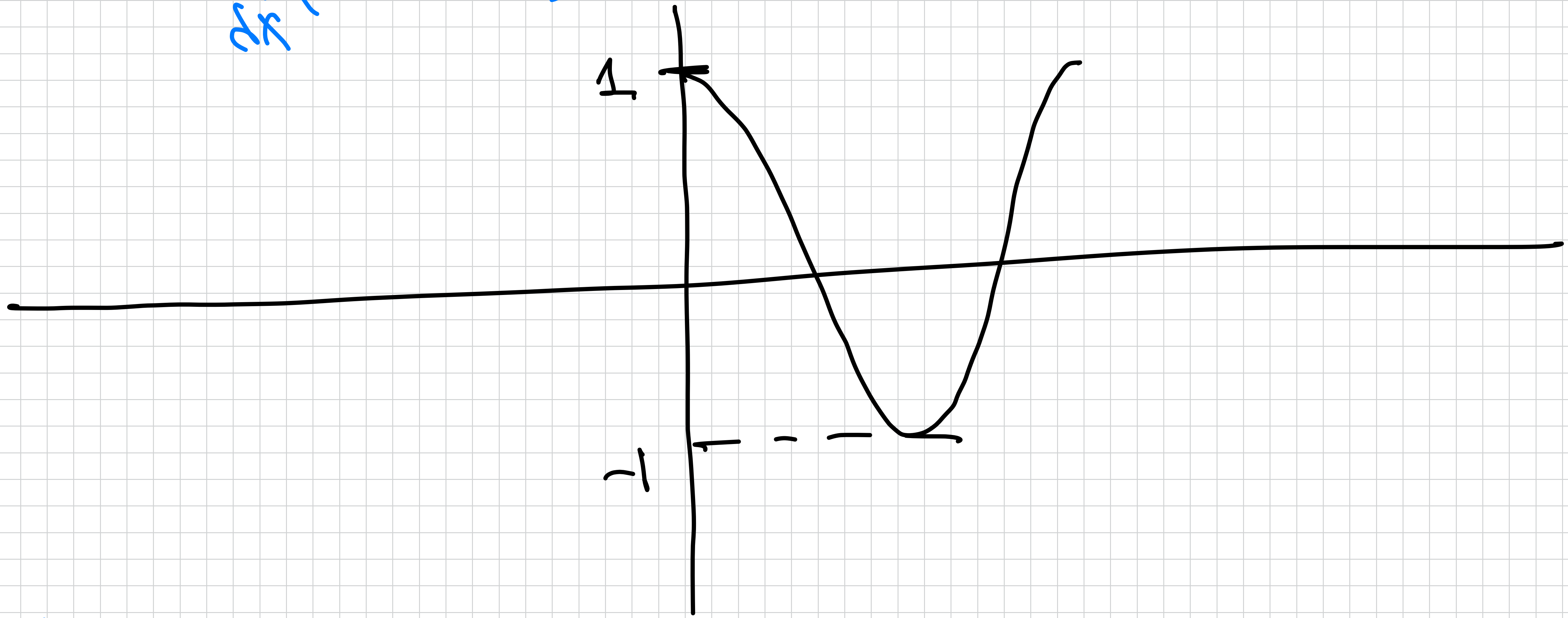
No V.A

$x \to \pm\infty$

No need to check end Behaviour as our Domain is $[-2\pi, 2\pi]$

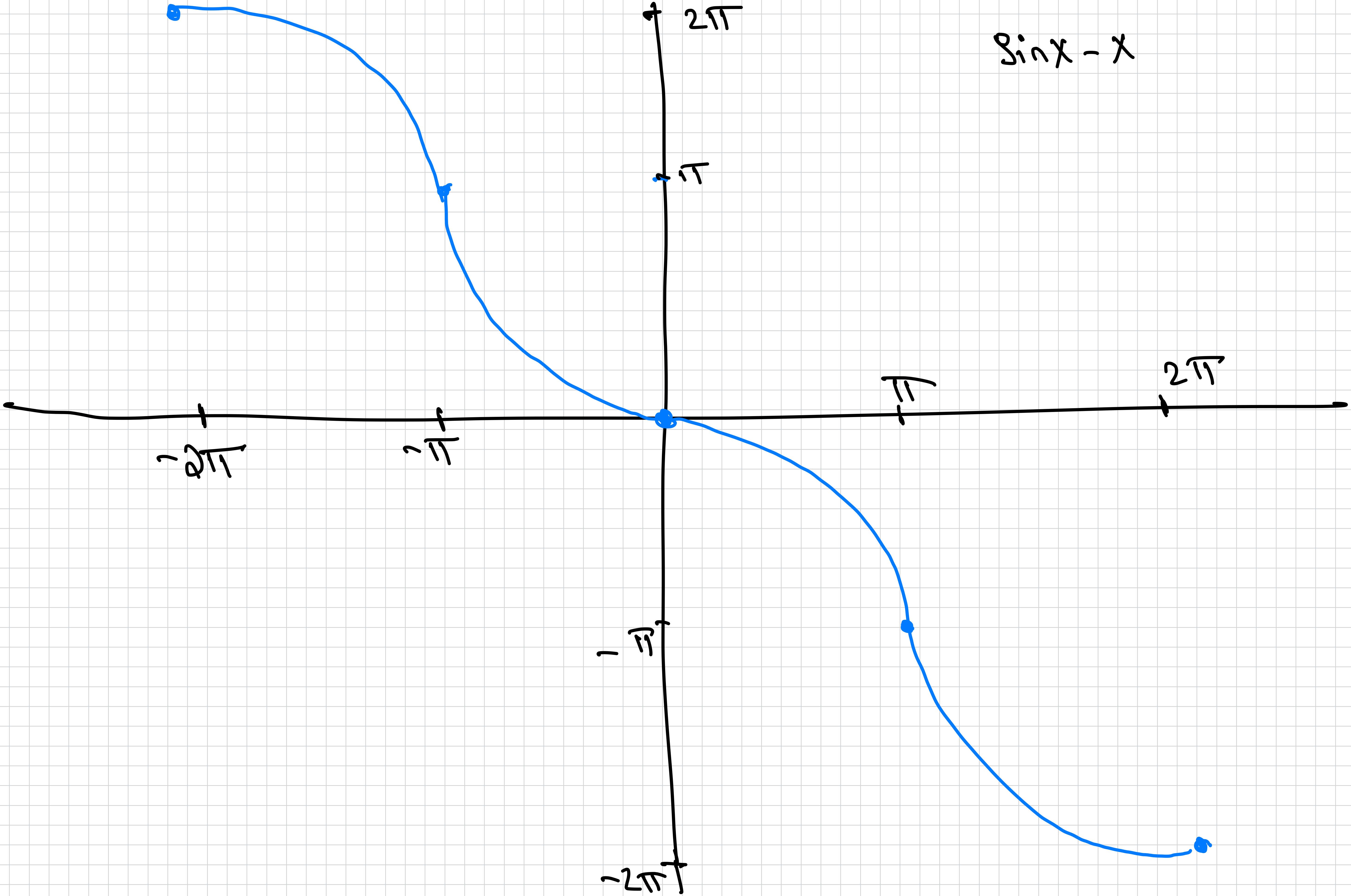
$f(x) = \sin x - x$

$f'(x) = \frac{d}{dx}(\sin x - x) = \cos x - 1$



$\cos x - 1 = 0 \rightsquigarrow x = 0, x = 2\pi$

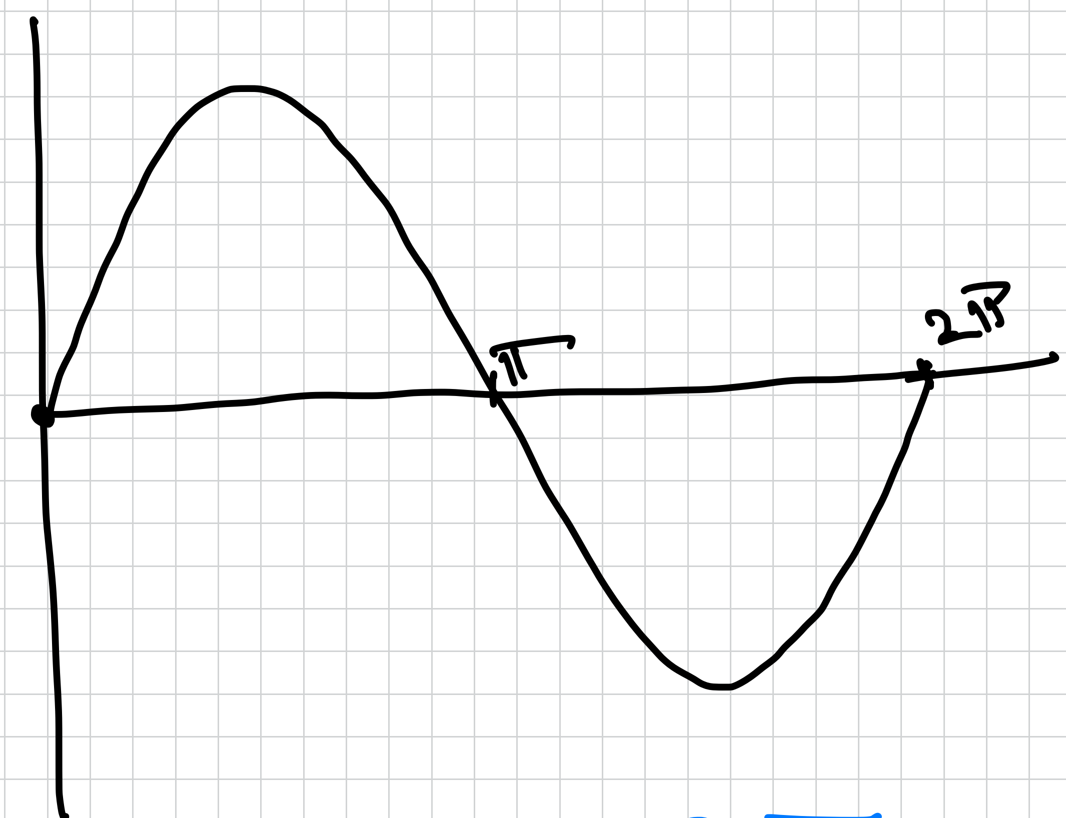
$\cos x - 1 < 0$ on $(0, 2\pi)$ $\longrightarrow$ $\underbrace{\text{Decreasing on } (0, 2\pi)}_{x=0 \text{ in the only } x\text{-intercept}}$ + Symmetry


sinx - x
2π
π
-2π
-π
π
2π
-π
-2π

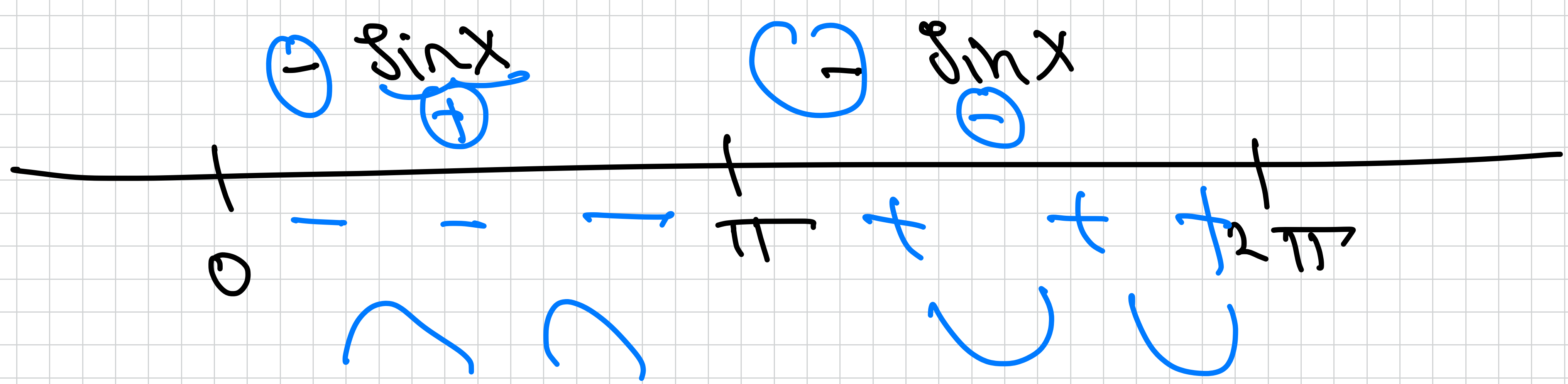
$f(x) = \sin x - x$

$f'(x) = \cos x - 1$

$f''(x) = \frac{d}{dx}(\cos x - 1) = -\sin x$

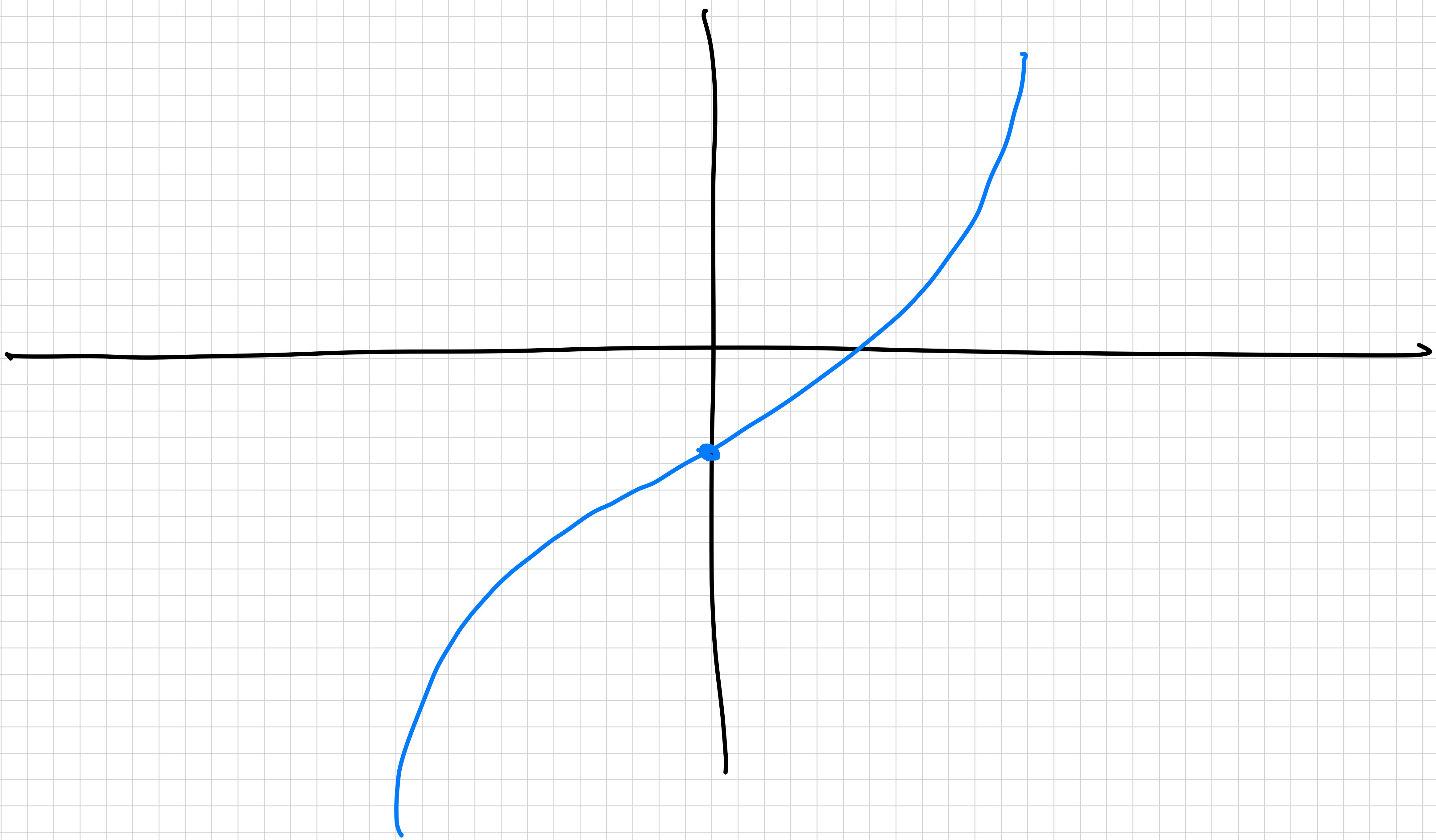


$f''(x) = 0 \rightarrow x = 0, \pi, 2\pi$



Given: $f'(x) = \underbrace{3x^2}$, Sketch $f(x)$

$$f''(x) = 6x \begin{cases} < 0 & \frown \\ > 0 & \smile \end{cases}$$



e.g: $f'(x) = (x+4)(x-1)(x-5)$, sketch $f(x)$

Critical! $(x+4)(x-1)(x-5) = 0 \rightarrow x = -4, \; x = 1, \; x = 5$

$f'(x) = (-)(-)(-)$ -4 $f'(x) = (+)(-)(-)$ 1 $f'(x) = (+)(+)(-)$ 5 $f'(x) = (+)(+)(+)$

$x = -4$ is a Local min

$x = 1$ is a local max

$x = 5$ is a Local min

$f'(x) = (x+4)(x-1)(x-5)$ } degree 3 polynomial

$f''(x) \rightsquigarrow$ quadratic.

$\Rightarrow$ atmost 2 points where its $= 0$

$\Downarrow$

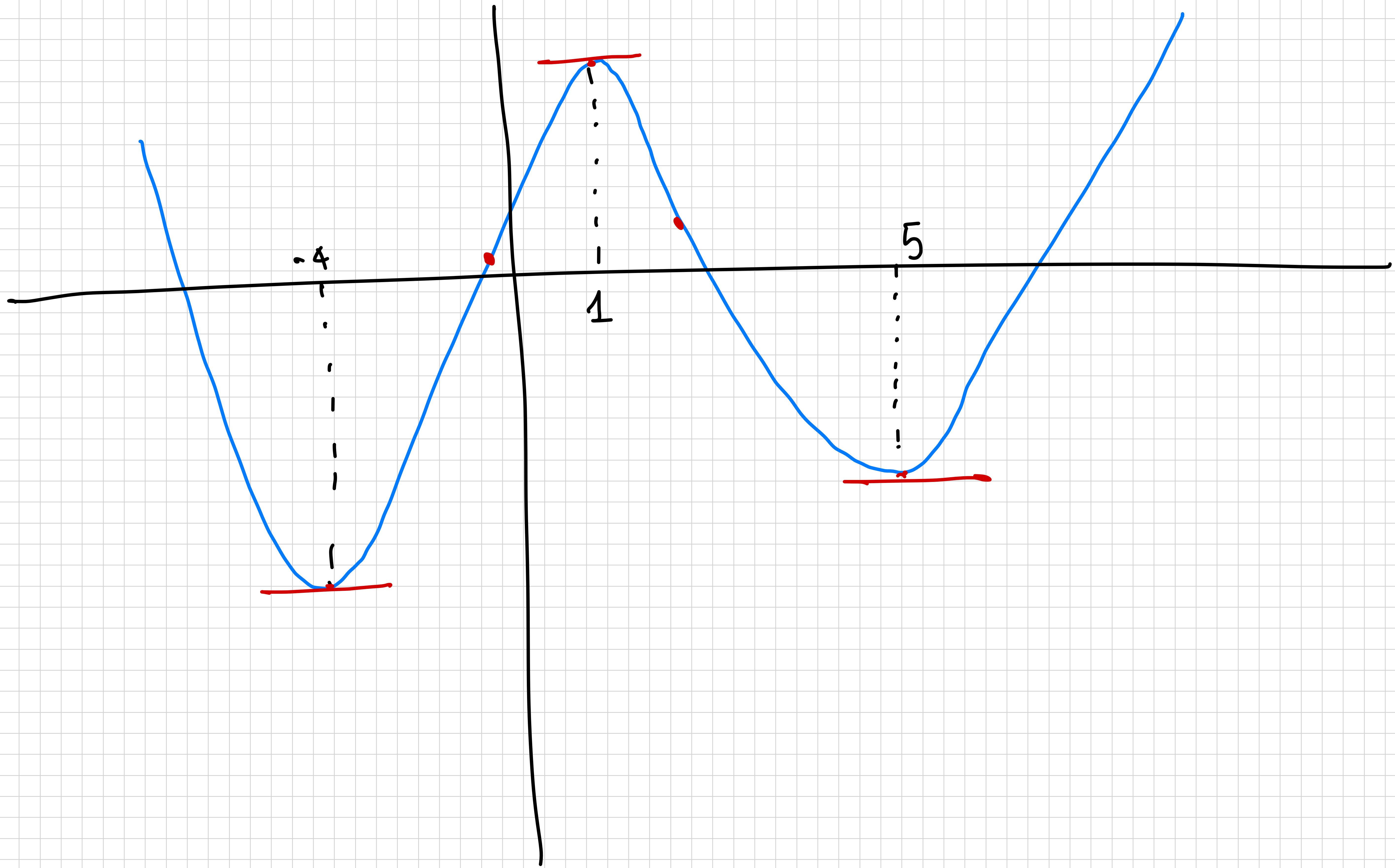
ATMOST 2 inflection points

exactly 2 inflection points.

Changes from c. down to c. up between $x=-4$ and $x=1$

Changes from c. up to c. down between $x=1$ and $x=5$

ATLEAST 2 inflection points



-4
1
5